Reply to the paper of V.D. Efros:

Comments in some recent papers on the Hyperspherical approach in few–body systems

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About a year ago V.D. Efros submitted a paper containing criticisms of our recent research activity on bound and scattering states of $A = 3$, 4 nucleons to a well-known international scientific review. That paper was not published and this unpleasant discussion was then avoided. Unfortunately, V.D. Efros still "feels himself obliged to discuss this issue in public" [1]. As a consequence, we are forced to reexamine the essence of the controversy and of the comments contained in ref. [1]. These are the motivations of the present paper.

Studies and calculations of few nucleon systems have been performed since long time using different techniques. In particular, the Hyperspherical Harmonic (HH) approach and Faddeev theory have been widely utilized. The theoretical grounds for both methods are now well established and most recent work has been concerned mainly with improving the accuracy of the numerical results.

A few years ago we started studying few nucleon systems using correlated Hyperspherical Harmonic bases, i.e. the Jastrow correlated basis CHH or pair correlated basis PHH functions. Only rather recently [2] results whose accuracy is close to that of the best available techniques [3] have been obtained by applying such correlated expansions (CHH or PHH) to the case of the $A = 3$ system interacting with realistic NN potentials. In ref. [2] the three–nucleon wave function is written as a sum of Faddeev-like amplitudes, each amplitude is decomposed in channels labelled by angular–spin–isospin quantum numbers and expanded in terms of correlated HH functions. This type of decomposition is quite standard, since it has been applied by many authors in a variety of papers. Consequently in ref. [2] the authors did not claim any original contribution to the procedure. On the other hand, since the problem is to obtain a truly accurate numerical solution of the problem, particular care must be taken in selecting the HH functions to be included in order to reduce the numerical calculations and obtain accurate results .

In our approach the expansion of the wave function is performed in two steps. In each channel all the correlated HH functions are considered corresponding to increasing values of the grand angular quantum number K until the desired accuracy is reached. The number of channels is then increased until complete stability is achieved for the calculated quantities. In this procedure, we select the maximum grand angular quantum number K_{max}

to have quite different values for the different channels so as to reflect its importance in the description of the wave function. The choice of the set of correlated HH functions is therefore rather straightforward, however the important point is in the adoption of suitable correlation factors. To give an idea of this point, in the case of realistic NN potentials the triton binding energy can be obtained with three accurate digits by using only 12 channels and 6 correlated HH functions per channel. This differs from the uncorrelated HH expansion as used in refs. [4–7] where all the basis elements of the selected channels having $K \leq K_{max}$ are included. It has to be stressed that with the uncorrelated expansion quite large values of K_{max} must be considered, as can be inferred from ref. [8].

This procedure was also followed when calculations were performed using the (uncorrelated) HH expansion [8]. Here again no claim to originality was advanced. The paper was a technical one, the purpose being to show that it is also possible with the uncorrelated HH basis to calculate the observables of the $A = 3$ bound state systems with great accuracy, superior to the present experimental values available. It is important to notice that the situation is quite different when $A > 3$, where the selection of the important channels and the HH components to be included in the expansion becomes a difficult problem [9–11].

The paper by Demin et al. [4] was cited in ref. [8] without any particular reference to their selection of the HH functions. Moreover, the calculations provided in ref. [5–7] were not accurate enough to fully justify the proposed selection. In fact, using the uncorrelated HH expansion, one has to overcome the difficulty of how to manage the very large number of basis functions required in the case of *hard* interactions, such as the MT(I-III) potential which has a $1/r$ repulsion, or of realistic interactions such as the Argonne type potentials. In ref. [8] very precise solutions were obtained for potentials of this kind, with an accuracy of six digits for the binding energy and four digits for other quantities of interest. In that context, we did not deem it appropriate to cite for comparison the papers [5–7] mentioned in the comments. The reason is that, in the latter papers, the most advanced application of the HH basis is contained in the paper by Mukhtarova [6], where a procedure similar to that later used by us in ref. [8] was applied. However, it is dangerous to refer to the paper [6] since something is wrong there. Presumably, that is why ref. [6] is not cited in any of the papers presenting accurate solutions of the three-nucleon problem (see, for example, ref. [12] and references cited therein). To be more explicit, the calculated triton binding energy for the SSC(C) interaction in ref. [6] is quoted to be $B = 7.608$ MeV, obtained by including HH functions up to $K_{max} = 34$ and with an extrapolated value of 7.65 MeV. The mixed symmetry percentage is presented as $P_{s'} = 1.15\%$. Our corresponding estimates (not presented in [8] since the SSC(C) interaction is rather soft and does not offer serious convergence problems) are $B = 7.5385$ MeV and $P_{s'} = 1.238\%$, respectively. The latter values confirm those already obtained in ref. [3] using the Faddeev technique. It is evident that the calculation of ref. [6] is not correct.

It may be worth noting that, as also stated in ref. [8], there is no problem in our approach with the uncorrelated HH basis in including three-nucleon interaction terms and in still obtaining very accurate results. This would not be possible using a numerical technique such as that proposed in refs. [4–7].

With regard to the treatment in refs. [2,13] of few-body reactions in the framework of the HH approach, it should be observed that our technique is based on correlated PHH functions. This is not a “minor difference”, as stated in ref. [1], but it is a crucial point since it is thereby possible to perform $n-d$ and $p-d$ scattering calculations with great accuracy. As a matter of fact, the PHH and Faddeev results have been successfully compared [14] and proposed as a benchmark for different approaches to the problem. On the other hand, the application of the HH expansion to scattering and reaction problems with a view to obtaining accurate results so as to make a fruitful comparison with the corresponding experimental data is as yet problematic. In fact, the convergence rate of the uncorrelated HH expansion for calculating $N-d$ scattering observables is for realistic potentials even slower than that observed in the $A=3$ bound state problem.

Moreover, the decomposition of the wave function as a sum of an asymptotic and an internal part is “natural”, as also stated by V.D. Efros and coll. in ref. [15]. In fact, the authors of ref. [15] did not deem it necessary to insert any specific references to earlier papers where the wave function was written in just that way. To be explicit, that decomposition was already used in various papers in the fifties and early sixties for studying $n-d$ scattering (see for example ref. [16]). The fact is that for a numerical application of the Kohn–Hulthèn variational principle, that decomposition of the trial wave function is rather obvious. For example, a very detailed review of the status at that time of the variational approach to $n-d$ scattering is reported in ref. [17]. There too, even though posterior to ref. [15], the decomposition of the trial wave function is reported without any particular comment.

To be noticed, that in our *first* paper on $N-d$ scattering [2], we have stated: “The variational approach based on the use of PHH correlated functions can be extended to investigate scattering states and in this section the application to the N–d scattering below the break–up threshold is discussed. Following the pioneering work of Delves [17] for realistic NN interactions, the wave function for a N–d scattering state will be written as

$$\Psi = \Psi_C + \Psi_A \ . \tag{1}$$

The first term $\Psi_C \ldots$”, etc. Therefore, it is “misleading” that the author of ref. [1] reported in his comment a citation of our subsequent paper [13] (which, moreover, is a Rapid Communication and therefore a rather short paper).

Last but not least the following remark appears to be worthy of consideration. A number of papers by the present authors devoted to the study of bound and scattering states of three and four nucleons have been published in international reviews. None of the referees has ever asked us to add any of the references cited by V.D. Efros.

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